ENERGY PRINCIPLE

The **Bernoulli Equation** and the **Energy Equation**

The **Bernoulli Equation** is applied for **a Steady flow, Incompressible, Irrotational & Inviscid** along a stream line

$$\left(\frac{p_1}{\gamma}+z_1+\frac{\overline{V}_1^{\,2}}{2g}\right)=\left(\frac{p_2}{\gamma}+z_2+\frac{\overline{V}_2^{\,2}}{2}\right)$$

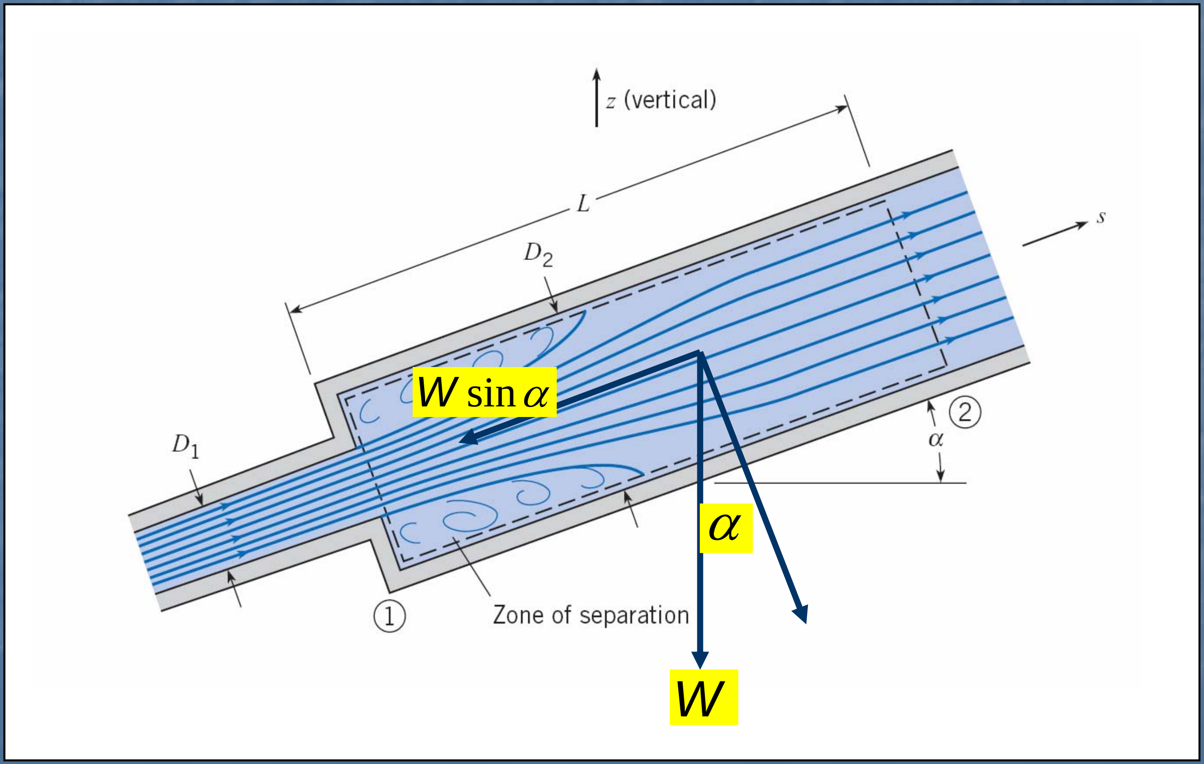
The **Steady Energy Equation** is applied for a **Steady flow Incompressible, Irrotational & Viscous flow** in a pipe with additional energy being added through a pump or extracted through a turbine

$$\left(h_P+\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part}=\left(h_T+\frac{p_2}{\rho g}+z_2+\alpha_2\frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part}+h_{Loss}$$

ENERGY PRINCIPLE

Application of the Energy Equation, Momentum and Continuity Principles in Combination

Abrupt Expansion



ENERGY PRINCIPLE

Required to find the Head Loss due to the expansion as a function of Flow Velocities in the two pipes?

Assumptions:

The pressure distribution at the change section (i.e. Section 1) is the same at the jet.

Applying the energy equation between point 1 & 2

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

The above Eqn. reduces to

$$\left(\frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(\frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

Eqn. (1)

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Applying the Momentum equation between point 1 & 2

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

Eqn. (2)

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$ (Steady Flow)

The above Eqn. (2) reduces to

$$\sum F_X = \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX} = \dot{m}V_2 - \dot{m}V_1$$

$$\sum F_X = p_1 A_2 - p_2 A_2 - \gamma A_2 L \sin\alpha = \rho V_2^{\,2} A_2 - \rho V_1^{\,2} A_1$$

$$But\ \sin\alpha = \frac{z_2 - z_1}{L}\ and\ divide\ Eqn.\,(2)\ by\ \gamma,\ we\ obtain,$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) = \frac{V_2^{\,2}}{g} - \frac{V_1^{\,2}}{g}\frac{A_1}{A_2}$$

Eqn. (3)

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From Continuity: $V_1 A_1 = V_2 A_2$ **and substituting in Eqn. (3), we obtain**

Rearranging Eqn. $\left(\frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(\frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$ **we have**

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) = \left(\frac{V_2^{\,2} - V_1^{\,2}}{2g}\right) + h_{loss}$$

Eqn. (4)

Combining Eqn. (3) & Eqn. (4), we obtain

$$i.e. \quad h_{loss} = \left(\frac{V_1^{\,2} - V_2^{\,2}}{2g}\right)$$

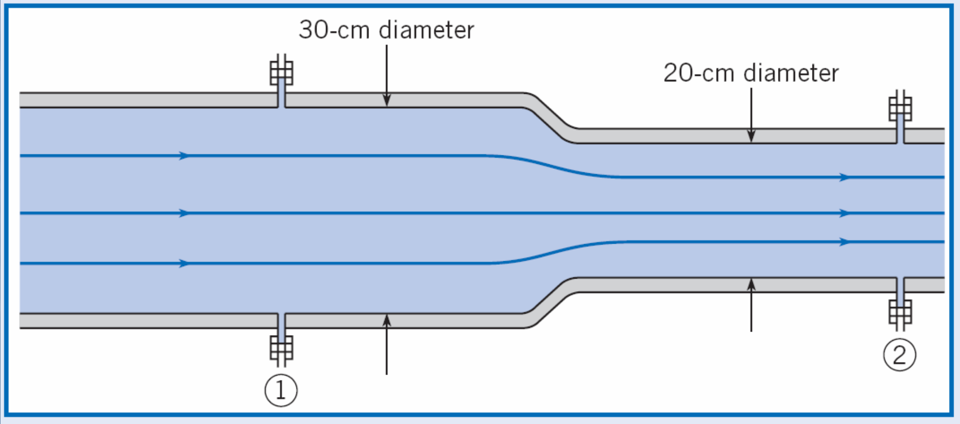
If a pipe discharge into a reservoir, then $V_2 = 0$

$$i.e. \quad h_{loss} = \left(\frac{V_1^{\,2}}{2g}\right)$$

Example (7.6)

Water flows through the contraction at a rate of 0.707 m^3/s. The head loss due to this particular transition is given by the empirical equation

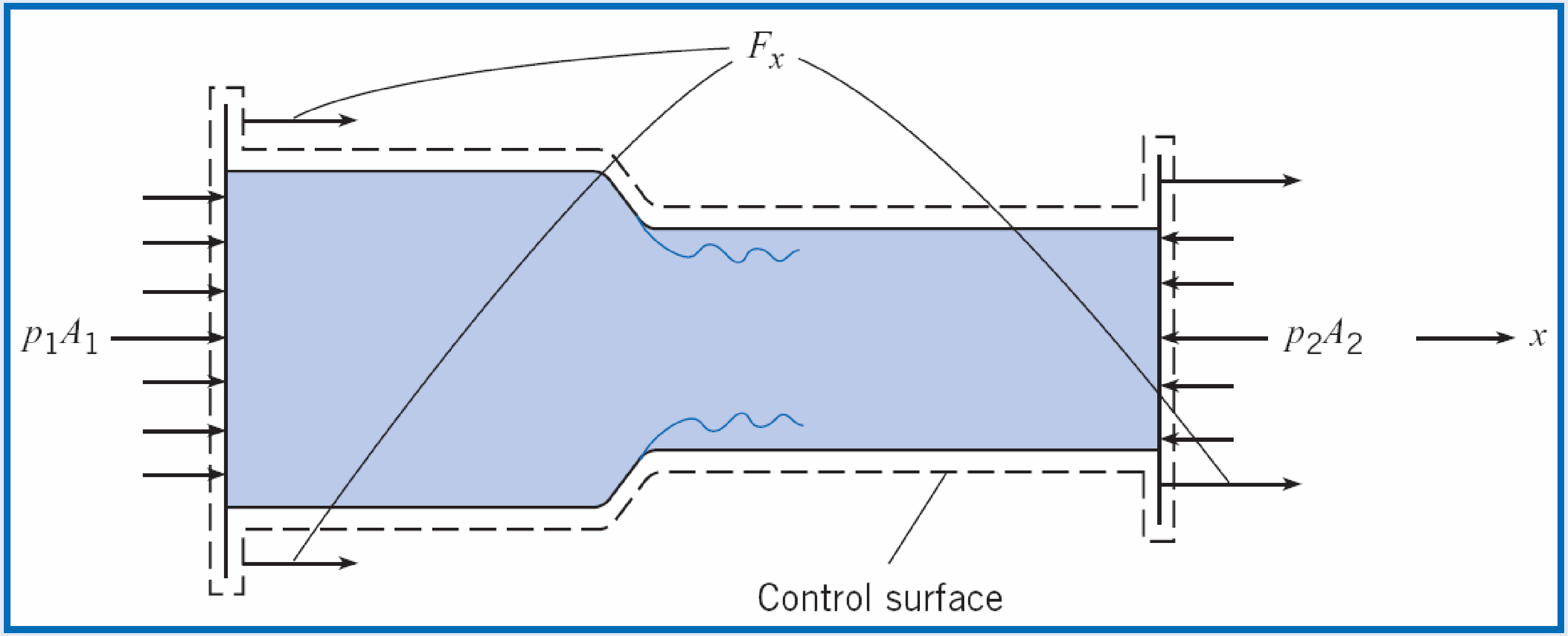
$$h_L = 0.1\frac{V_2^2}{2g}$$



Given:

$$p_1 = 250\,kPa, \quad h_{Loss} = 0.1\frac{V_2^{\,2}}{2g}, \quad \dot{Q} = 0.707\,m^3/s, \quad \alpha_1 = \alpha_2 = 1$$

Find: Horizontal force (F_x) required to hold the transition in place?



Applying the Momentum equation between point 1 & 2

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho dQ + \sum_{cs}(\dot{m}v)_{outX} - \sum_{cs}(\dot{m}v)_{inX}$$

Eqn. (1)

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ **Flow is steady**

The above Eqn.(1) reduces to $\sum F_x = \sum_{cs}(\dot{m}v)_{outX} - \sum_{cs}(\dot{m}v)_{inX} = \dot{m}_2 V_2 - \dot{m}_1 V_1$

$$\sum F_X = p_1A_1 - p_2A_2 + F_X = \rho V_2{}^2 A_2 - \rho V_1{}^2 A_1$$ **Eqn. (2)**

$$F_X = p_2A_2 - p_1A_1 + \rho Q(V_2 - V_1) \qquad AS\ \dot{Q} = A_1V_1 = A_2V_2$$ **Eqn. (3)**

Applying the energy equation between point 1 & 2

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V_1}^2}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V_2}^2}{2g}\right)_{Mech.part} + h_{Loss}$$ **Eqn. (4)**

$$Since \quad z_1 = z_2, \quad h_p = 0, \quad h_T = 0, \quad \alpha_1 = \alpha_2 = 1$$

i.e. Eqn (4) reduces to

$$\left(\frac{p_1}{\rho g} + \frac{\overline{V_1}^2}{2g}\right)_{Mech.part} = \left(\frac{p_2}{\rho g} + \frac{\overline{V_2}^2}{2g}\right)_{Mech.part} + h_{Loss}$$

$$p_2 = p_1 - \gamma\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L\right)$$ **Eqn. (5)**

Substituting Eqn. (5) into Eqn. (3), we have

$$F_x = \rho Q(V_2 - V_1) + A_2\left[p_1 - \gamma\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L\right)\right] - p_1 A_1$$

$$V_1 = \frac{Q}{A_1} = \frac{0.707}{(\pi/4)\times 0.3^2} = 10\ \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.707}{(\pi/4)\times 0.2^2} = 22.5\ \text{m/s}$$

$$h_L = \frac{0.1V_2^2}{2g} = \frac{0.1\times 22.5^2}{2\times 9.81} = 2.58\ \text{m}$$

Then

$$F_x = 1000\times(0.707)(22.5-10) + \frac{\pi}{4}\times 0.2^2$$
$$\times\left[250{,}000 - 9810\left(\frac{22.5^2}{2\times 9.81} - \frac{10^2}{2\times 9.81} + 2.58\right)\right]$$
$$-250{,}000\times(\pi/4)\times 0.3^2$$
$$= -8.15\ \text{kN} \qquad \triangleleft$$

Thus a force of 8.15 kN must be applied in the negative x direction to hold the transition in place for the given conditions.

END OF LECTURE (5)